EXERCISES [MAI 3.6-3.7]

TRIGONOMETRIC CIRCLE – RADIANS – TRIANGLES II

SOLUTIONS

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A. Paper 1 questions (SHORT)

TRIGONOMETRIC CIRCLE AND RADIANS

1. (a) (i) $\frac{\pi}{9}$ (ii) $\frac{\pi}{10}$ (iii) 3π

(b) (i) 10° (ii) 36° (iii) 450°.

2. (a)

	A	B	C	D
in degrees	30°	150°	210°	330°
in radians	$\frac{\pi}{6}$	$\frac{5\pi}{6}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$

(b)

	A	B	C	D
in degrees	40°	140°	220°	320°
in radians	$\frac{2\pi}{9}$	$\frac{7\pi}{9}$	$\frac{11\pi}{9}$	$\frac{16\pi}{9}$

3.

	In degrees	
2nd period backwards	$-720° \le \theta < -360°$	-690°
1st period backwards	$-360° \le \theta < 0°$	-330°
1st period	$0° \le \theta < 360°$	30°
2nd period	$360° \le \theta < 720°$	390°
3rd period	$720° \le \theta < 1080°$	750°

in radians	
$-4\pi \le \theta < 2\pi$	$-\frac{23\pi}{6}$
$-2\pi \le \theta < 0$	$-\frac{11\pi}{6}$
$0 \le \theta < 2\pi$	$\frac{\pi}{6}$
$2\pi \le \theta < 4\pi$	$\frac{13\pi}{6}$
$4\pi \le \theta < 6\pi$	$\frac{25\pi}{6}$

4. (a) $A = \pi - 0.7 - 0.5 = 1.94159 = 1.94$

(b) $5 / \sin 0.5 = AC / \sin 0.7 \Rightarrow AC = 6.71864 = 6.72$

(c) $5 / \sin 0.5 = BC / \sin 1.94159 \Rightarrow BC = 9.72038 = 9.72$

(d) $\text{Area} = 1/2 \times 5 \times 9.72038 \sin 0.7 = 15.7$

ARCS AND SECTORS (IN RADIANS)

5. (a) $l_{MINOR} = (10)(1.5) = 15$ $\quad l_{MAJOR} = (10)(2\pi - 1.5) = 20\pi - 15$

(b) $A_{MINOR} = \frac{1}{2}(10)^2(1.5) = 75$ $\quad A_{MAJOR} = \frac{1}{2}(10)^2(2\pi - 1.5) = 200\pi - 75$

(c) $P_{MINOR} = l_{MINOR} + 2r = 35$ $\quad P_{MAJOR} = l_{MAJOR} + 2r = 20\pi + 5$

6. (a) $l = r\theta$ or ACB = 2 × OA= 30 cm

(b) $A\hat{O}B$ (obtuse) = $2\pi - 2$ $\Rightarrow$ Area = $\frac{1}{2}\theta r^2 = \frac{1}{2}(2\pi - 2)(15)^2$= 482 cm^2 (3 sf)

7. Perimeter = $5(2\pi - 1) + 10 = (10\pi + 5)$ cm (= 36.4, to 3 sf)

8. (a) **METHOD 1**

cosine rule AB = $\sqrt{3.9^2 + 3.9^2 - 2(3.9)(3.9)\cos 1.8}$ = 6.11(cm)

METHOD 2

using right-angled triangles $\sin 0.9 = \frac{x}{3.9} \Rightarrow x = 3.9 \sin 0.9$

AB = $2x$ = 6.11 (cm)

METHOD 3

sine rule $\frac{\sin 0.670...}{3.9} = \frac{\sin 1.8}{AB} \Rightarrow$ AB = 6.11 (cm)

(b) $A\hat{O}B = 2\pi - 1.8$ (= 4.4832) $\Rightarrow$ $A = \frac{1}{2}(3.9)^2(4.4832...)$ = 34.1 (cm^2)

9. (a) $A = \frac{1}{2}r^2\theta \Rightarrow 27 = \frac{1}{2}(1.5)r^2 \Rightarrow r^2 = 36 \Rightarrow r = 6$ cm

(b) $l = r\theta = 1.5 \times 6 = 9$ cm

10. (a) $3\pi = r\frac{2\pi}{9} \Rightarrow r = 13.5$ (cm)

(b) perimeter = $2r + l$ = $27 + 3\pi$ (cm) (= 36.4)

(c) area = $\frac{1}{2} \times 13.5^2 \times \frac{2\pi}{9} = 20.25\pi$ (cm^2) (= 63.6)

11. $A = \frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}r^2\theta = \frac{4}{3}\pi$ $\qquad l = r\theta \Rightarrow r\theta = \frac{2}{3}\pi$

Divide the two relations: $r = 4$, and so $\theta = \frac{\pi}{6}$

12. (a) Area = $\frac{1}{2}r^2\theta = \frac{1}{2}(15^2)(2) = 225$

(b) Area ΔOAB = $\frac{1}{2}15^2 \sin 2 = 102.3$

Area = 225 – 102.3 = 122.7 = 123 (3 sf)

(c) $AB^2 = 15^2 + 15^2 - 2\times15\times15\times\cos 2 \Rightarrow$ AB = 25.24412… = 25.2

arc AB = 15×2 =30

Perimeter = 30 + 25.2 = 55.2

13. (a) $20 = 2r + r\theta \Rightarrow \theta = \dfrac{20-2r}{r}$

(b) $A = \dfrac{1}{2}r^2\left(\dfrac{20-2r}{r}\right)\ \left(=10r-r^2\right)$

$10r - r^2 = 25 \Rightarrow r = 5$ cm

14. **METHOD A**

Area of triangle = $\dfrac{1}{2}r^2\theta$ Area of segment = $\dfrac{1}{2}r^2(\theta - \sin\theta)$

$\dfrac{1}{2}r^2\theta = 3 \times \dfrac{1}{2}r^2(\theta - \sin\theta) \Rightarrow \theta = 3(\theta - \sin\theta) \Rightarrow 4\sin\theta = 3\theta \Rightarrow \theta = 1.28$

METHOD B

Area of triangle = 3/ 4 Area of sector

$\dfrac{1}{2}r^2\theta = 3/4 \times \dfrac{1}{2}r^2\sin\theta \Rightarrow \theta = 3/4\ \sin\theta \Rightarrow \theta = 1.28$

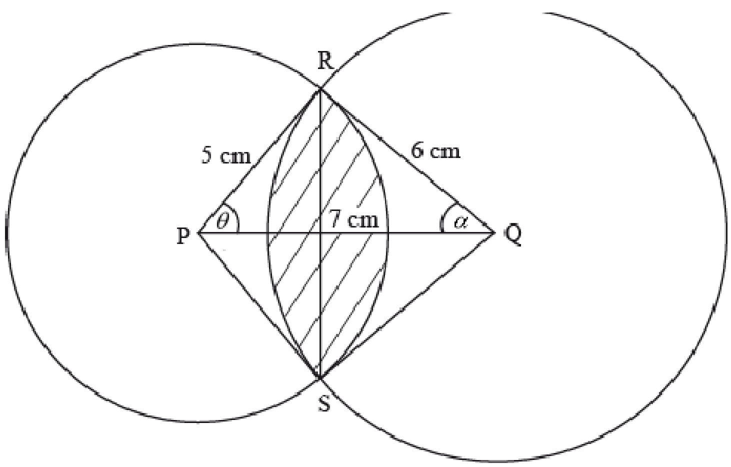
15. Area sector OAB $= \dfrac{1}{2}(5)^2(0.8) = 10$

$\cos 0.8 = \text{ON}/5 \Rightarrow \text{ON} = 5\cos 0.8\ \ (=3.483...)$

Area of $\Delta\text{AON} = \dfrac{1}{2}\text{ON} \times 5\times\sin 0.8 = 6.249...$

Shaded area $= 10 - 6.249.. = 3.75$

16.



$\cos\theta = \dfrac{5^2+7^2-6^2}{2\times5\times7} = \dfrac{25+49-36}{70} = \dfrac{38}{70} \Rightarrow \theta = 0.997$

$\Rightarrow 2\theta = 1.99...$

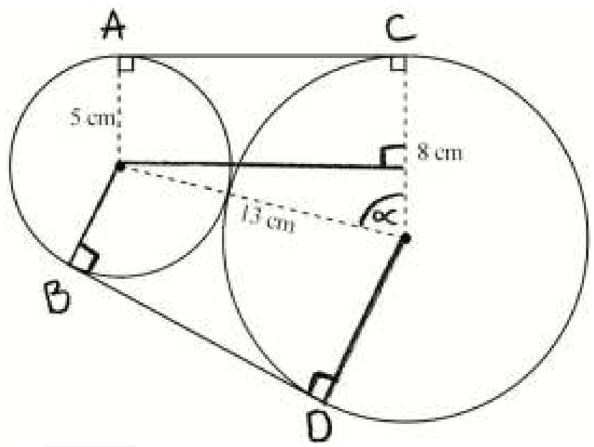
$\cos\alpha = \dfrac{7^2+6^2-5^2}{2\times7\times6} = \dfrac{49+36-25}{84} = \dfrac{60}{84} = 0.775$

$\Rightarrow 2\alpha = 1.55...$

Required area $= \dfrac{1}{2}5^2(1.99 - \sin 1.99) + \dfrac{1}{2}6^2(1.55 - \sin 1.55)$

$= 23.4\text{ cm}^2$

17.



$AC = BD = \sqrt{13^2 - 3^2} = 12.64\ldots$

$\cos\alpha = \dfrac{3}{13} \Rightarrow \alpha = 1.337\ldots(76.65\ldots°)$

arc length $AB = 5(\pi - 2\times 0.232\ldots)$ $(=13.37\ldots)$

arc length $CD = 8(\pi + 2\times 0.232\ldots)$ $(=28.85\ldots)$

length of string = 13.37…+ 28.85…+2×12.64… = 67.5 (cm)

TRIANGLES II – AMBIGUOUS CASE

18.

$0° \le \theta \le 90°$	$\sin\theta$
30°	**0.500**
40°	**0.643**
50°	0.766
37°	0.600

$90° \le \theta \le 180°$	$\sin\theta$
150°	**0.500**
140°	**0.643**
130°	0.766
143°	0.600

19. (a) $B\hat{A}C = 105°$

(b) $B\hat{D}A = 135°$, $B\hat{A}D = 15°$

(c) $BC^2 = 9^2 + 5^2 - 2\times9\times5\times\cos105° \Rightarrow BC = 11.4$

$BD^2 = 9^2 + 5^2 - 2\times9\times5\times\cos15° \Rightarrow BD = 4.36$

20. $\sin C = \dfrac{c\sin A}{a} = \dfrac{5\times 0.5}{3}$

$\hat{C} = 56.4°$ and then $\hat{B} = 93.6°$ **or** $\hat{C} = 123.6°$ and then $\hat{B} = 26.4°$

21. Sine rule: $\dfrac{\sin 35}{4} = \dfrac{\sin\hat{B}}{6.5} \Rightarrow \sin\hat{B} = \dfrac{6.5\sin 35}{4} \Rightarrow \sin\hat{B} = 0.932$

Hence $\hat{B} = 68.8$ or $\hat{B} = 180 - 68.8 = 111.2$

If $\hat{B} = 68.8$, then $\hat{C} = 180 - 35 - 68.8 = 76.2$

Sine rule again: $\dfrac{\sin 35}{4} = \dfrac{\sin 76.2}{AB} \Rightarrow AB = 6.77$

If $\hat{B} = 111.2$, then $\hat{C} = 180 - 35 - 111.2 = 33.8$

Sine rule again: $\dfrac{\sin 35}{4} = \dfrac{\sin 33.8}{AB} \Rightarrow AB = 3.88$

22. **Method 1**:

Using the sine rule: $\frac{\sin C}{6} = \frac{\sin 30°}{3\sqrt{2}} \Rightarrow \sin C = \frac{1}{\sqrt{2}} \Rightarrow C = 45°, 135°.$

If $C = 45°, B = 105°$ $\quad \frac{3\sqrt{2}}{\sin 30°} = \frac{BC}{\sin 105°} \Rightarrow BC = 8.20$ cm

If $C = 135°, B = 15°$ $\quad \frac{3\sqrt{2}}{\sin 30°} = \frac{BC}{\sin 15°} \Rightarrow BC = 2.20$ cm

Method 2:

Using the cosine rule: $AC^2 = 6^2 + BC^2 - 2(6)(BC)\cos 30° \Rightarrow 18 = 36 + BC^2 - 6\sqrt{3}\,BC$

(In fact, this is a quadratic equation: $BC^2 - (6\sqrt{3})BC + 18 = 0$)

BC = 8.20 cm or BC = 2.20 cm.

23.

METHOD 1

$\frac{\sin 31}{3} = \frac{\sin BAC}{5}$

$\sin BAC = \frac{5\sin 31}{3}$

$\angle BAC = 59.137° \text{ or } 120.863°$

$\angle ACB = 89.863° \text{ or } 28.137°$

$\frac{3}{\sin 31} = \frac{AB}{\sin 89.863} \qquad \frac{3}{\sin 31} = \frac{AB}{\sin 28.137}$

$AB = \frac{3\sin 89.863}{\sin 31} \text{ or } AB = \frac{3\sin 28.137}{\sin 31}$

$AB = 5.82 \qquad AB = 2.75$

METHOD 2

$3^2 = 5^2 + AB^2 - 2 \times AB \times BC \times \cos 31°$

$0 = AB^2 - 10\,AB\cos 31° + 16$

AB = 5.82 or 2.75

24. $\frac{\sin(A\hat{C}B)}{20} = \frac{\sin 50°}{17} \Rightarrow \sin(A\hat{C}B) = \frac{20\sin 50°}{17}$

$A\hat{C}B = 64.3°$ OR $A\hat{C}B = 180° - 64.3° = 115.7° = 116$ (3 sf)

(a) In Triangle 2, $A\hat{C}B > 90°$, $A\hat{C}B = 116$

(b) In Triangle 1, $A\hat{C}B = 64.3°$

$B\hat{A}C = 180° - (64.3° + 50°) = 65.7°$

Area = $\frac{1}{2}(20)(17)\sin 65.7° = 155\ (\text{cm}^2)$

25. (a) substitution into the formula for the area of a triangle

$\sin C = \frac{8}{13.6} \Rightarrow \hat{C} = 36.031° \approx 36°$ or $180° - 36°$

$A\hat{C}B = 144°$ (2.51 radians)

(b) cosine rule: $(AB)^2 = 5^2 + 13.6^2 - 2(5)(13.6)\cos 143.968... \Rightarrow AB = 17.9$

26. Area of a triangle = $\frac{1}{2} \times 3 \times 4 \sin A = 4.5 \Rightarrow \sin A = 0.75$

$A = 48.6°$ and $A = 131°$

B. Paper 2 questions (LONG)

27. (a) (i) $x = 5$ (ii) $y_{max} = 144$

(b) (i) $z = 10 - x$

(ii) $z^2 = x^2 + 6^2 - 2x(6)\cos Z$

(iii) $100 - 20x + x^2 = x^2 + 36 - 12x\cos Z$

$12x\cos Z = 20x - 64 \Rightarrow \cos Z = \dfrac{20x - 64}{12x} \Rightarrow \cos Z = \dfrac{5x-16}{3x}$

(c) $A = \dfrac{1}{2} \times 6 \times x \times \sin Z = 3x\sin Z \Rightarrow A^2 = 9x^2\sin^2 Z$

(d) $A^2 = 9x^2\sin^2 Z = 9x^2(1 - \cos^2 Z) = 9x^2 - 9x^2\cos^2 Z = 9x^2 - 9x^2\left(\dfrac{5x-16}{3x}\right)^2$

$A^2 = 9x^2 - (25x^2 - 160x + 256) = -16x^2 + 160x - 256$

(e) (i) $A_{max} = 12$ (ii) Isosceles

28. (a) $A = \dfrac{1}{2}x\,3x\sin\theta$ so $\sin\theta = \dfrac{4.42}{3x^2}$

(b) $\cos\theta = \dfrac{x^2 + (3x)^2 - (x+3)^2}{2 \times x \times 3x} = \dfrac{3x^2 - 2x - 3}{2x^2}$

(c) (i) Substituting the answers from (a) and (b) into the identity $\cos^2\theta = 1 - \sin^2\theta$

$$\left(\frac{3x^2 - 2x - 3}{2x^2}\right)^2 = 1 - \left(\frac{4.42}{3x^2}\right)^2$$

(ii) $x = 1.24, 2.94$

$\theta = 1.86$ radians or $\theta = 0.171$

29. (a) $\dfrac{AD}{\sin 0.8} = \dfrac{4}{\sin 0.3} \Rightarrow AD = 9.71$ (cm)

(b) $OAD = \pi - 1.1 = (2.04)$

EITHER $OD^2 = 9.71^2 + 4^2 - 2 \times 9.71 \times 4 \times \cos(\pi - 1.1) \Rightarrow OD = 12.1$

OR. $\dfrac{OD}{\sin(\pi - 1.1)} = \dfrac{9.71}{\sin 0.8} = \dfrac{4}{\sin 0.3} \Rightarrow OD = 12.1$

(c) area $= 0.5 \times 4^2 \times 0.8 = 6.4$

(d) area of triangle OAD: . $A = \dfrac{1}{2} \times 4 \times 12.1 \times \sin 0.8 = 17.3067$

(**OR** $A = \dfrac{1}{2} \times 4 \times 9.71 \times \sin 2.04 = 17.3067$ **OR** $A = \dfrac{1}{2} \times 12.1 \times 9.71 \times \sin 0.3 = 17.3067$

area ABCD $= 17.3067 - 6.4 = 10.9$ (cm^2)

30. (a) cosine rule: $4^2 = 3^2 + 2^2 - 2\times3\times2\cos A\hat{O}P$

$A\hat{O}P = 1.82$ (radians)

(b) $A\hat{O}B = 2\pi - 2\times1.82 = 2\pi - 3.64 = 2.64$ (radians)

(c) (i) Area of sector PAEB = $\frac{1}{2}\times4^2\times1.63 = 13.04$ (cm^2)

(ii) Area of sector OADB = $\frac{1}{2}\times3^2\times2.64 = 11.9$ (cm^2)

(d) (i) Area AOBE = Area PAEB – Area AOBP = 13.04 – 5.81 = 7.23

(ii) Area shaded = Area OADB – Area AOBE = 11.9 – 7.23 = 4.67

31. (a) $\frac{6}{\sin A} = \frac{\frac{7\sqrt{2}}{2}}{\sin 45^\circ} \Rightarrow \sin A = 6\times\frac{\sqrt{2}}{2}\times\frac{2}{7\sqrt{2}} = \frac{6}{7}$

(b) (i) $B\hat{D}C + B\hat{A}C = 180^\circ$

(ii) $\sin A = \frac{6}{7}$ => A = 59.0° or 121° => $B\hat{C}D = 180^\circ - (121^\circ + 45^\circ) = 14.0^\circ$

(iii) $\frac{BD}{\sin 14^\circ} = \frac{\frac{7\sqrt{2}}{2}}{\sin 45^\circ}$ => BD = 1.69

(c) $\frac{\text{Area}\,\Delta BCD}{\text{Area}\,\Delta BAC} = \frac{\frac{1}{2}BD\times6\sin45}{\frac{1}{2}BA\times6\sin45} = \frac{BD}{BA}$